N-factors and Design. Are We Expecting Too Much?

Proposal of tasks for the next workshop

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1st AIAA Transition Modelling and Prediction Workshop

21-22 January 2021, Virtual Event

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Part 1

N-factors and Design. Are We Expecting Too Much?

Transition prediction with two N-factors

Geometry
Pressure distribution
Suction distribution

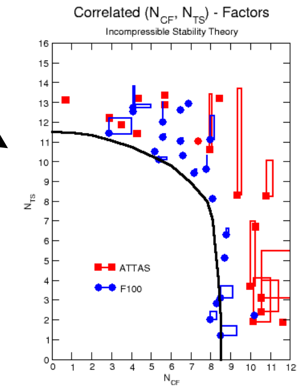
↓

Boundary layer calculation

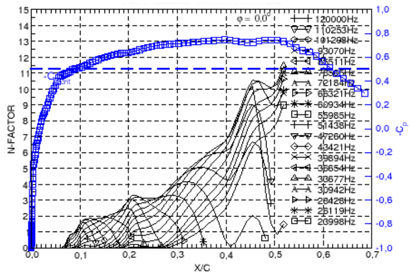
↓

Stability calculations

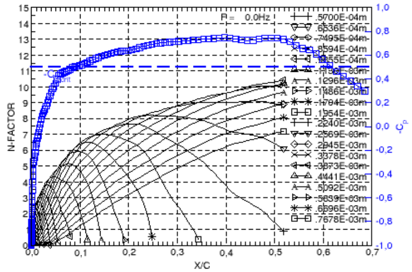
FT or WTT Correlation



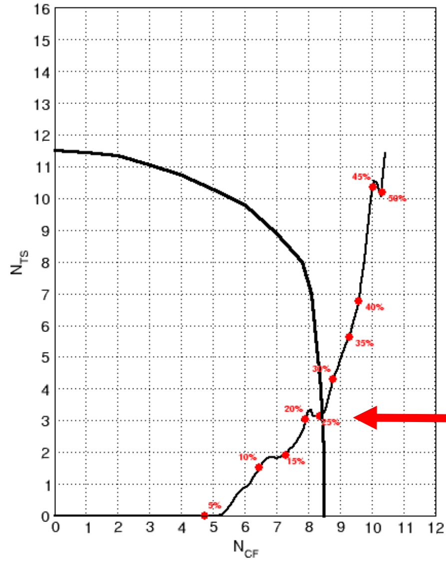
TS N-factors



CF N-factors



Correlation curve



Estimated transition location

Mode ansatz for linear stability theory (LST)

$$q'(x,y,z,t) \;=\; \hat{q}(z)\; e^{i(\alpha_r x + \beta_r y - \omega_r t)}\; e^{-(\alpha_i x + \beta_i y)}\; e^{\omega_i t}$$

3D: 6 quantities $\alpha_r, \beta_r, \omega_r, \alpha_i, \beta_i, \omega_i$

2D: only 4 quantities because
flow direction = wave propagation direction = amplification direction

2D: problem closed. There are sufficiently many equations for the N-factor computation

Follow modes with prescribed frequency

3D: problem not closed! One[1] additional condition is needed!

Follow modes
with prescribed frequency and an additional prescribed quantity!

Remark 1: 6 – 4 = 2, i.e. two additional conditions are needed.
However, the amplification direction is treated equivalently in all codes

Possible choices for the missing condition in 3D

Infinitely long, swept wing: **Flow quantities constant in spanwise direction**	➔ **spanwise wave number**	N_β
Stripe pattern for CF-instability: **Stationary waves with nearly constant wave length**	➔ **wave length**	N_{CF}
Incompressible LST: **Maximal amplification of TS-waves in direction of inviscid flow**	➔ **propagation direction**	N_{TS}
Optimize amplification rate over wave length or propagation direction	➔ **envelope method**	N

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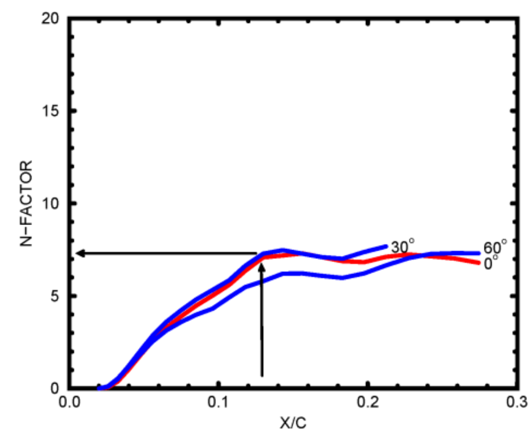
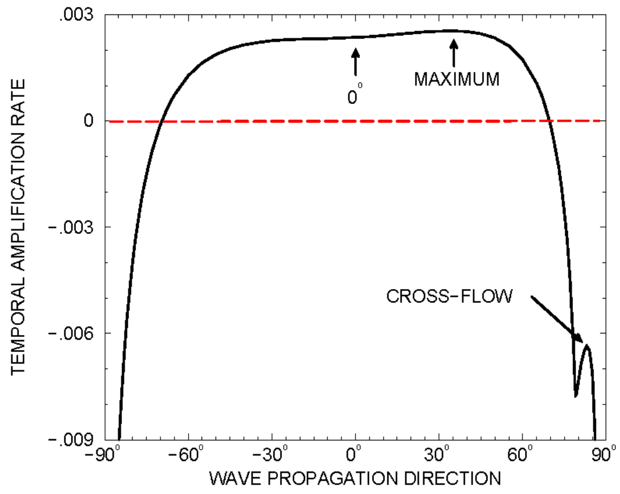
Our procedure for Tollmien-Schlichting modes

Prescribed frequency and 0º-propagation direction



Evaluation of F100 flight tests with compressible LST showed $N_{\varphi=0} \sim N_{EE}$ ➔ use $N_{EE} = N_{\varphi=0}$, i.e. consider only the 0º-direction



5000Hz mode on the F100 glove at 12%
Variation of the propagation direction

- **Maximal rate only 1% higher than rate for 0º**
- **Rates asymmetric in propagation direction**

Remark A:	**Fokker 100 flight test:**	**M = 0.50 – 0.80, Re = 17 – 30 Million**
	TELFONA ETW test:	**M = 0.76 – 0.80, Re = 15 – 23 Million**
	A320 HLFC flight test:	**M = 0.76 – 0.80, Re = 17 – 25 Million**
	S1MA HLFC w/t test:	**M = 0.50 – 0.82, Re = 13 – 23 Million**

We have no correlation for Mach > 0.82

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Our procedure for crossflow modes

0 Hz frequency and prescribed wave length

Use N_{CF} or N_{β}

Consider only 0Hz-frequency
even though travelling CF-waves exhibit larger amplification

Experiments:

Stationary waves dominate transition
in low-turbulence environment

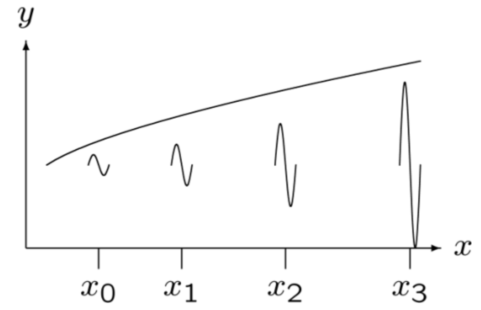
Remark B: **Fixed spanwise wave numbers and fixed wave lengths result in very similar correlated N-factors.**

Remarks on validity of LST

- **The LST equations are only valid for modes with a small amplitude so that squares* (and cubes) can be neglected.**
- **The local amplification rates computed with LST are valid near the neutral point.**
- **Experiments show that the local amplification rates obtained with LST are good approximations for amplitudes that are not too large ($A / U_{edge} < 0.1$).**
- **The initial phase of mode growth is described relatively well by LST.**
- **LST cannot be correct in the non-linear regime where mode interactions cause very strong amplification that finally leads to transition.**

* $(0.001)^2 = 0.000\ 001$
$(0.001)^3 = 0.000\ 000\ 001$

Application of e^N-method



Step 1: Compute local amplification rates with LST

Step 2: Integrate to obtain a global amplification rate, for example in 2D.

$$\begin{aligned} A/A_0 &= e^{-\alpha_i(x_1)[x_1-x_0]} \cdot e^{-\alpha_i(x_2)[x_2-x_1]} \ldots e^{-\alpha_i(x_n)[x_n-x_{n-1}]} \\ &= e^{-\int_{x_0}^{x} \alpha_i(x)dx} \\ &= e^{N(x)} \end{aligned}$$

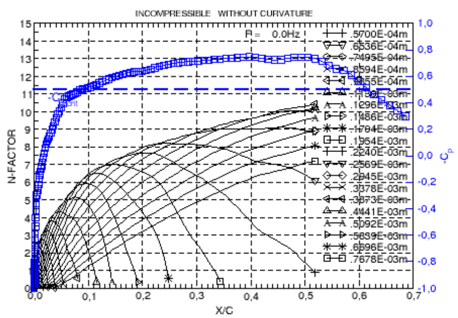
The N-factor is the logarithm of the global amplification rate
compute one N-factor curve

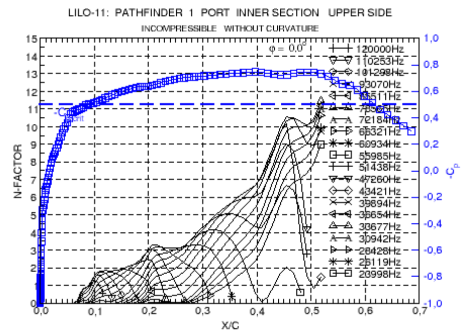
Remark C: Because of linearity, the N-factor does not depend on the initial amplitude of the mode.

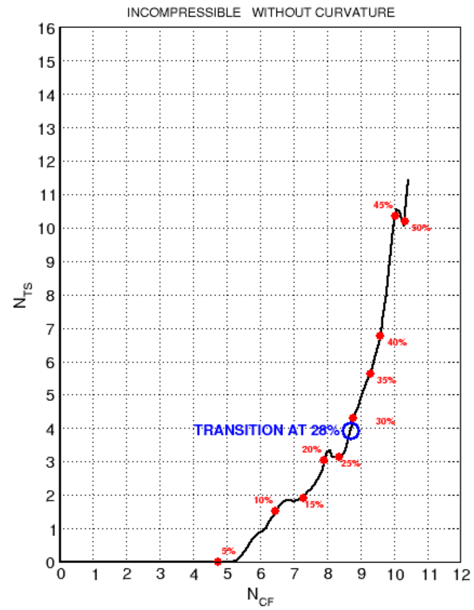
Remark D: Assuming that initial amplitudes are in the order of 10^{-3} and that the amplification rates obtained with LST are a good approximations up to 10^{-1}, then the N-factor should be a good approximation up to $N = \ln(100) \approx 5$.

Remark E: If the non-linear region is short compared to the region of linear amplification, the e^N-method will predict a good approximation of the transition location.

Application of e^N-method: make a correlation







Step 3: **Compute N_{CF}-factor curves for several cross-flow modes to obtain the N_{CF}-envelope.**

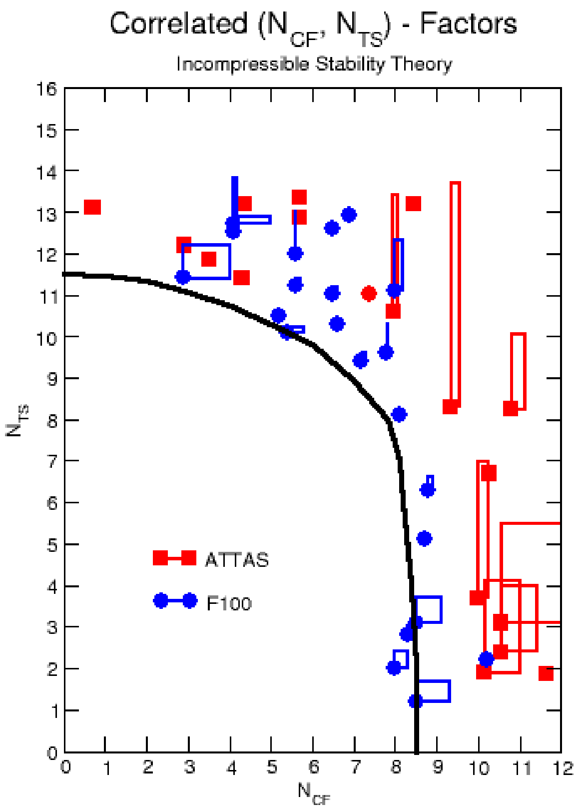
Compute N_{TS}-factor curves for several Tollmien-Schlichting modes to obtain the N_{TS}-envelope.

The (N_{CF} ,N_{TS})-pairs of both envelopes form a curve with X/C as parameter.

Step 4c: **Correlation: If the transition location is known the corresponding point on the (N_{CF} ,N_{TS})- curve is marked**.

Application of e^N-method: example for a correlation

The N-factor correlation must be based on experiments for similar flow conditions, for example, similar Re, similar Mach



Example:
Correlation based on VFW614 ATTAS and ELFIN Fokker F100 flight tests for incompressible stability theory

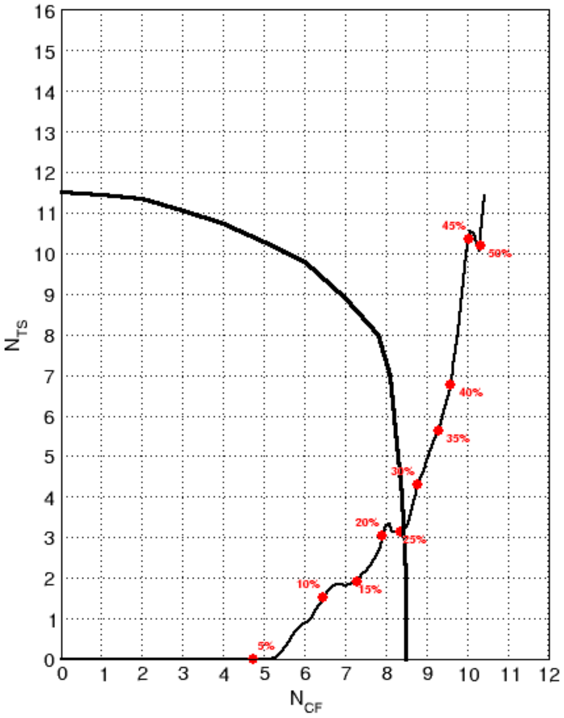
Remark F: **This correlation curve is intentionally pessimistic, because it is placed on the inside of the correlation band.**

Application of e^N-method: transition prediction

Perform steps 1 to 3

Application of correlation

Step 4p: Transition prediction:
Obtain the transition location as intersection of the (N_{CF}, N_{TS})- curve with the correlation curve



Examples for re-applications on F100 cases

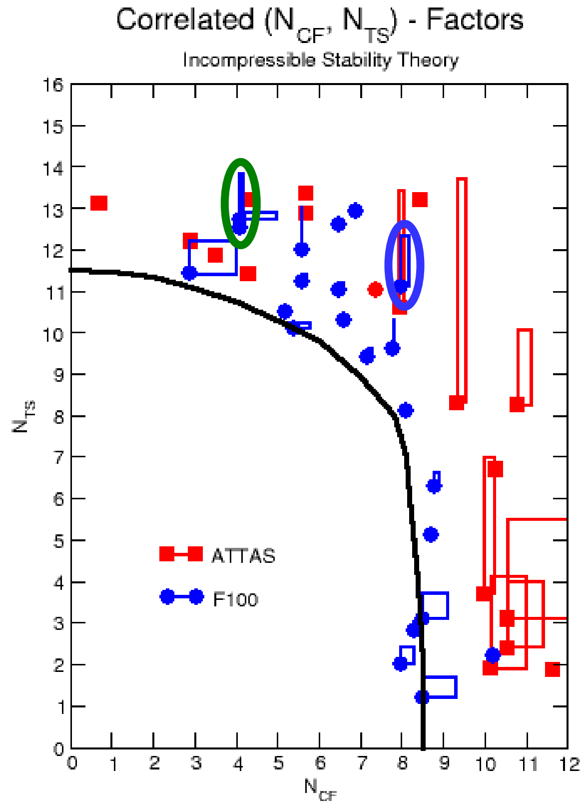
To demonstrate the limits of the e^{N}-method,
it is re-applied to some F100 cases used for correlation.

Because the correlation curve is placed inside of the correlation points in the (N_{CF},N_{TS})-diagram, the transition estimation is pessimistic, i.e. the predicted transition is upstream of the measured one.

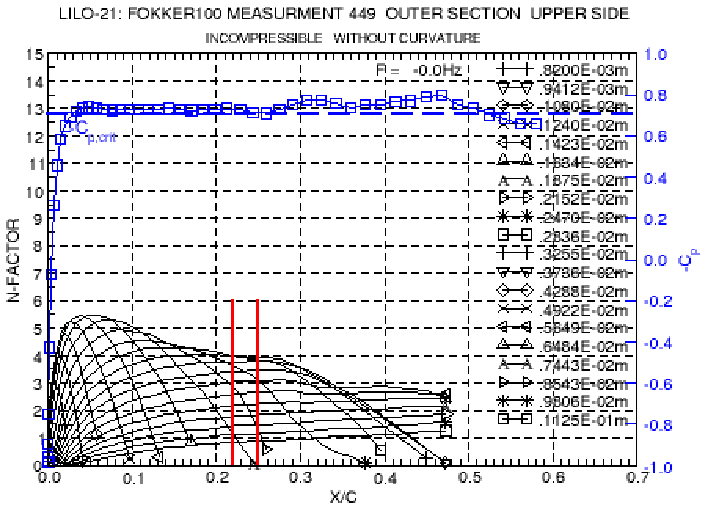
There continues to be discussion on the right correlation.

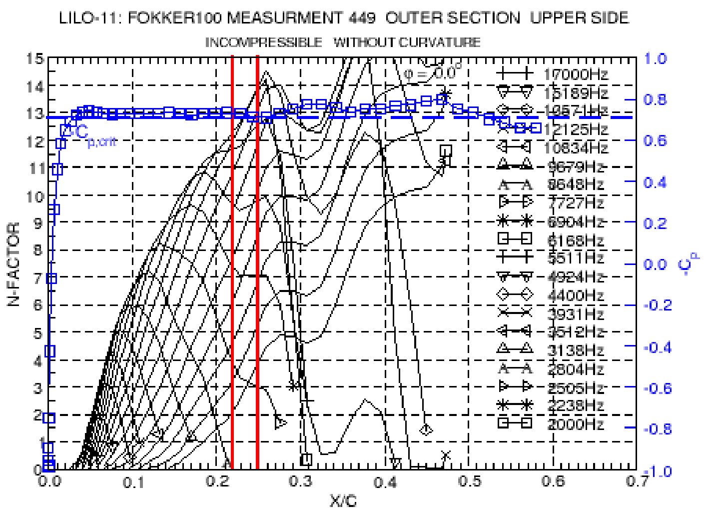
Case A: clear Tollmien-Schlichting transition

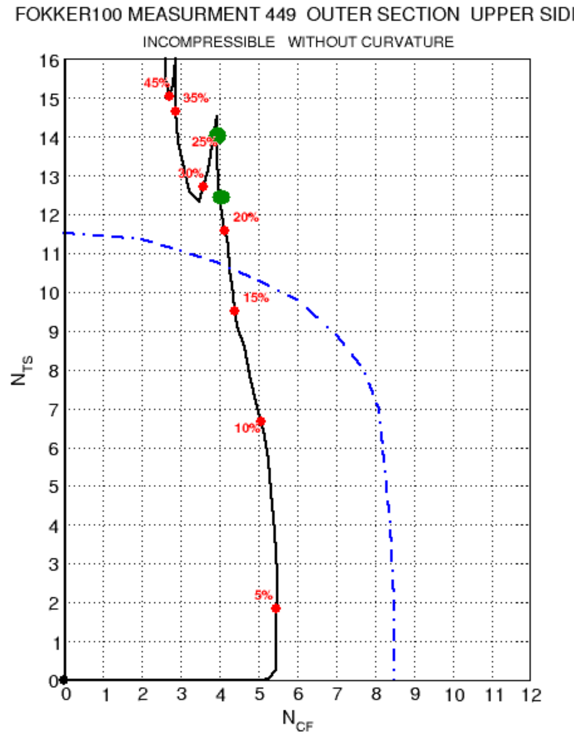
Case B: TS-transition with large CF-amplification



Case A: a clear Tollmien-Schlichting transition

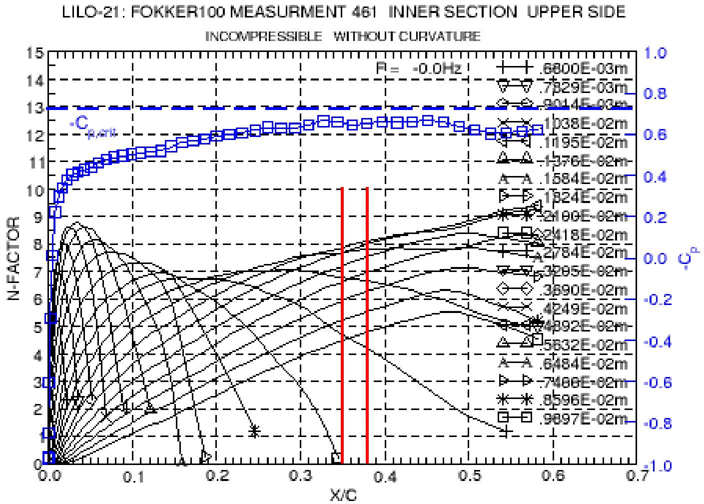


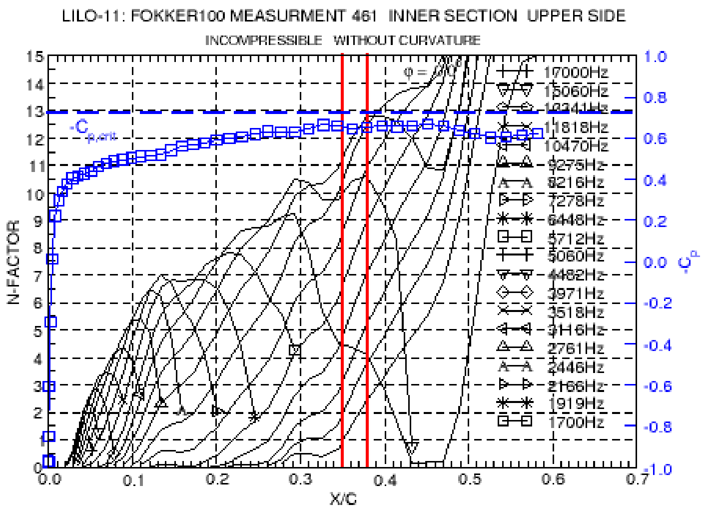


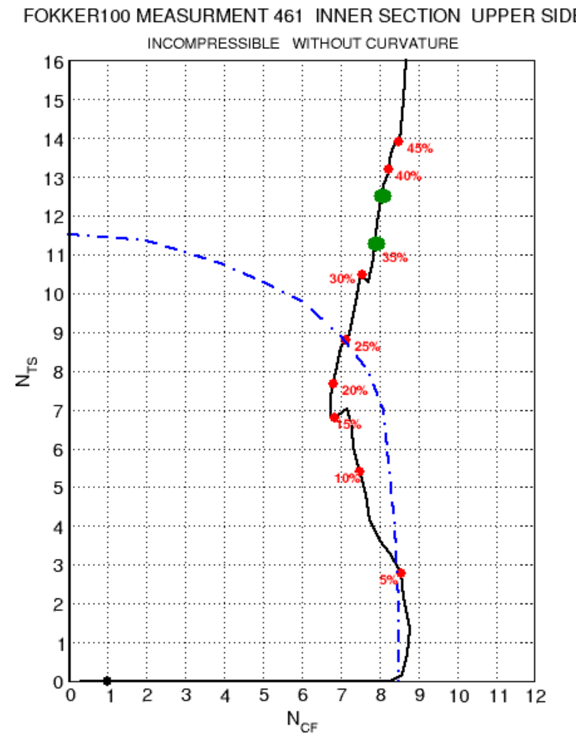


Observed transition: 22 - 25%
Predicted transition: 17%

Case B: TS transition with large CF-amplification







Observed transition: 35 - 38%

Predicted transition: 2% / 25%

Remark G: **In design, we should avoid such cases because we cannot safely predict transition with the e^N-method!**

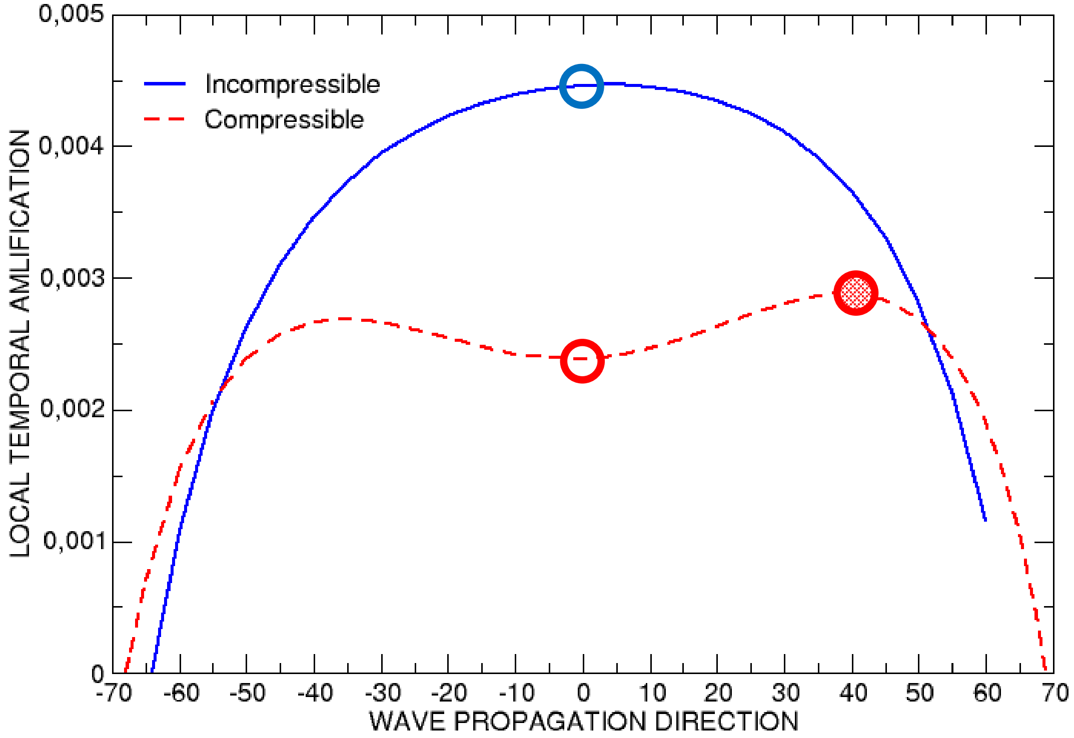
Airfoil F: incompressible vs. compressible LST

T. Streit, A. Seitz, P. Kunze, S. Hein: "NLF Potential of Laminar Transonic Long Range Aircraft."
AIAA Aviation 2020 Forum, June 15-19, 2020, Virtual Event

Airfoil F	**Local chord length:**	**1 m**	**Mach:**	**0.83**
	Leading edge sweep:	**32º**	**Re:**	**35 million**
	Trailing edge sweep:	**21.4º**		

Local amplification rates
at one station in the boundary layer
for several wave propagation directions

X/C	**0.163**
Local Mach	**1.13**



Airfoil F: incompressible vs. compressible LST

Compressible stability theory:

Cross-flow:	**0 Hz**
Tollmien-Schlichting:	**0º direction**

Compressible stability theory: most amplified Tollmien-Schlichting mode in 40º-direction

Cross-flow:	**0 Hz**
Tollmien-Schlichting:	**40º direction**

Incompressible stability theory

Cross-flow:	**0 Hz**
Tollmien-Schlichting:	**0º direction**

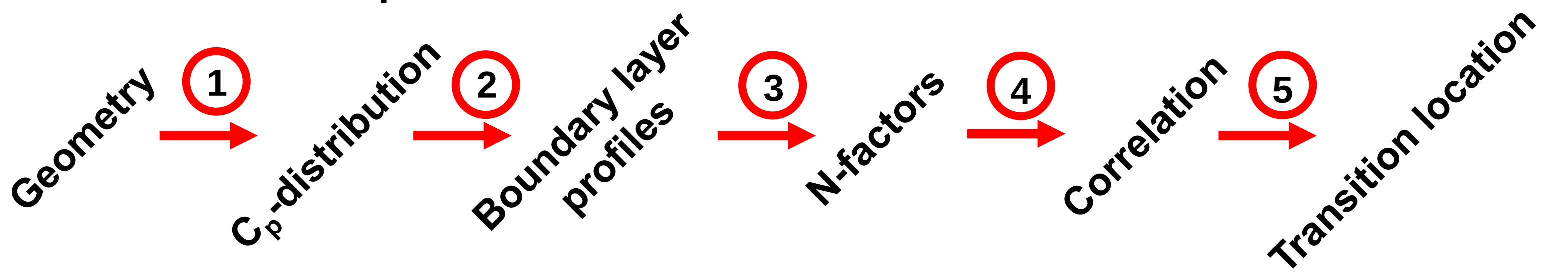
Transition results will be presented tomorrow by Thomas Streit

Part 2

Proposal for a comparison of linear stability results within the next transition prediction workshop

Proposal of tasks for a future workshop

Tool chain for transition prediction with the e^N-method

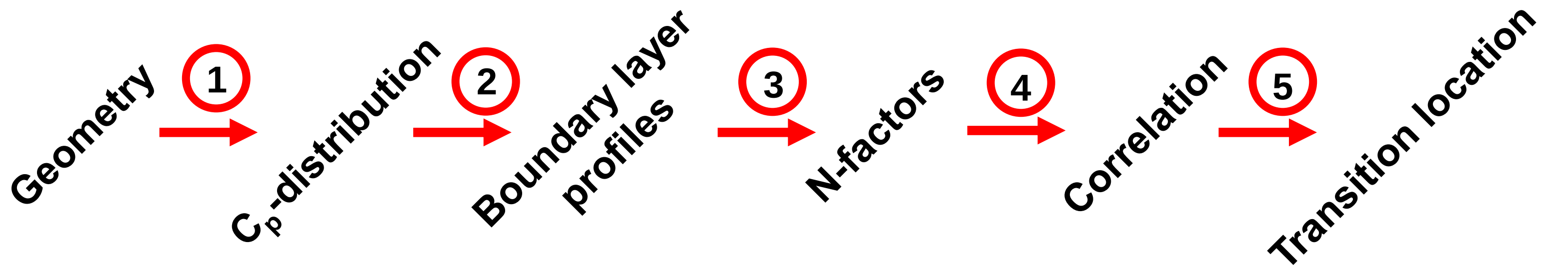


In the present workshop, the complete tool chain is considered

In case of different results, it is difficult to find out which step is contributing how much to those differences

Therefore, we propose to consider each step separately

Proposal of tasks for the next workshop



Which steps would be suitable for a workshop?

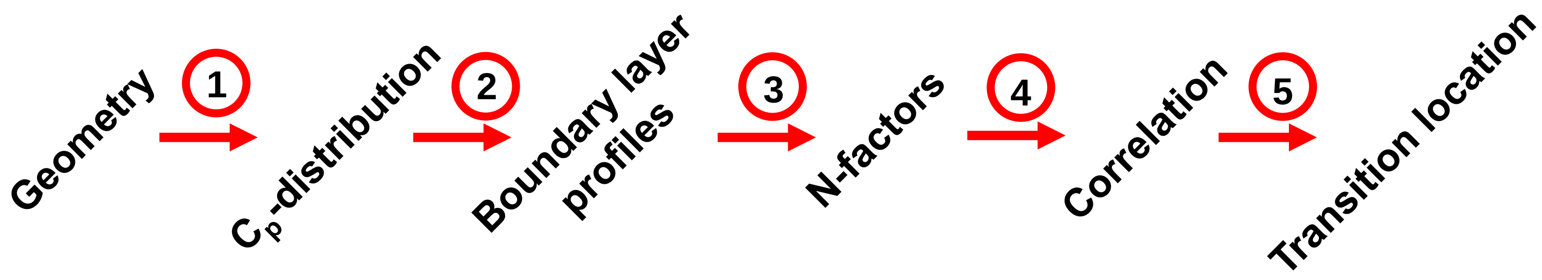
Our proposal:

Step 2: **Comparison of boundary layer profiles**
computed for a given geometry and a given pressure (and suction) distribution
with a boundary-layer or an Navier-Stokes method

We propose to consider a 2D, an infinite swept, and a conical wing geometry
to be defined by the partners

All participants should use the same input
and compute the boundary layer profiles for given locations

Proposal of tasks for the next workshop



Step 3: Comparison of stability results computed for a given input, i.e. with the same boundary layer profiles

All participants should use given input boundary layer profiles* for their stability codes

Again, we propose to consider a 2D, an infinite swept, and a conical geometry to be defined by the partners

Local amplification rates as well as N-factors should be compared

*** The profiles could be provided in the EUROTRANS format which has been developed for such a comparison**

Proposal of tasks for the next workshop

**Use for workshop selected cases from (TS and more CF)
(Geometry, measured pressure distribution, infra-red images)**

Fokker 100 flight test: M = 0.50 - 0.80, Re = 17 - 30 Million

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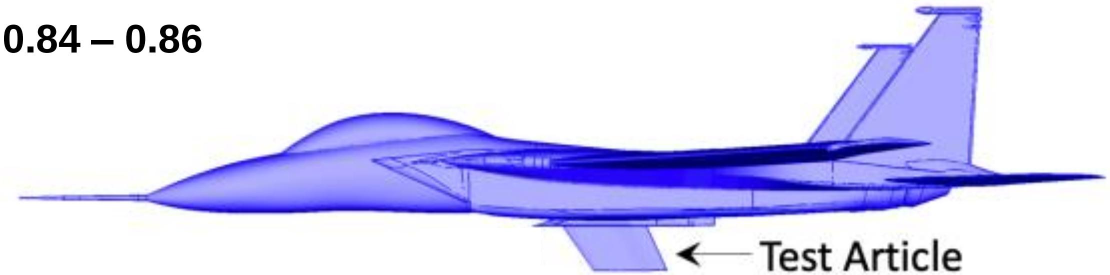
S1MA HLFC w/t test: M = 0.50 - 0.82, Re = 13 - 23 Million

A320 HLFC flight test: M = 0.76 - 0.80, Re = 17 - 25 Million

Test cases could be defined after clearance

CATNLF flight test: M = 0.84 – 0.86



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Appendix

Linear stability results for Airfoil F

Airfoil F: incompressible vs. compressible LST

T. Streit, A. Seitz, P. Kunze, S. Hein: "NLF Potential of Laminar Transonic Long Range Aircraft."
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Local amplification rates
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for several wave propagation directions

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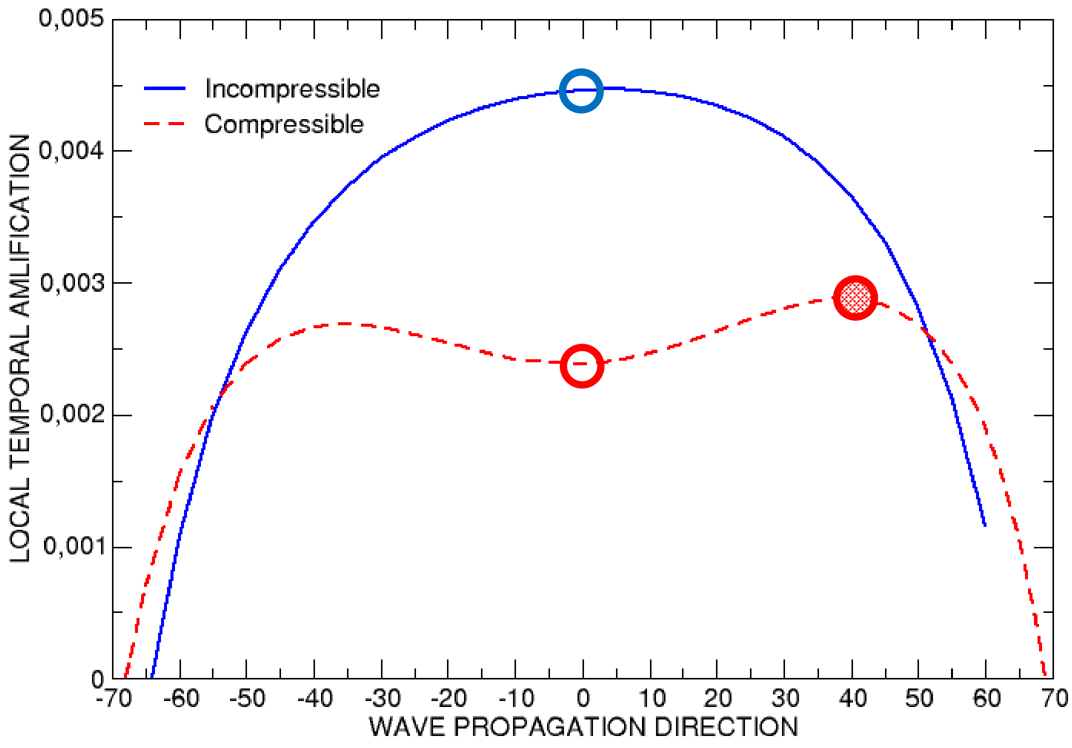
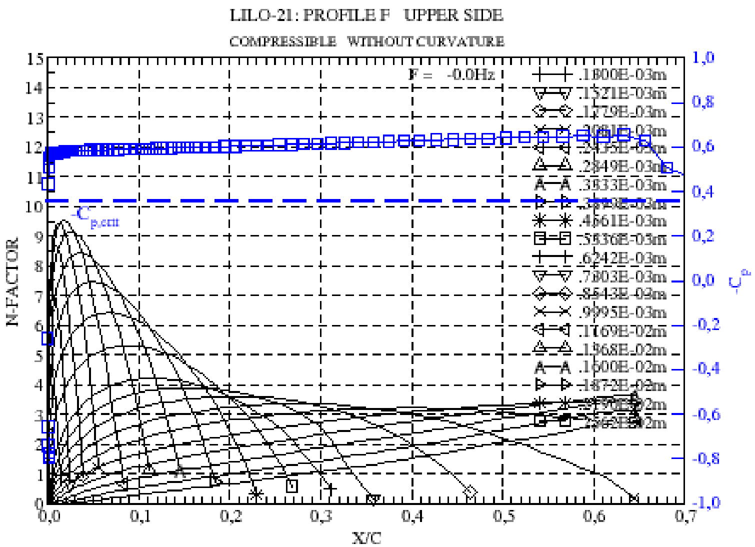
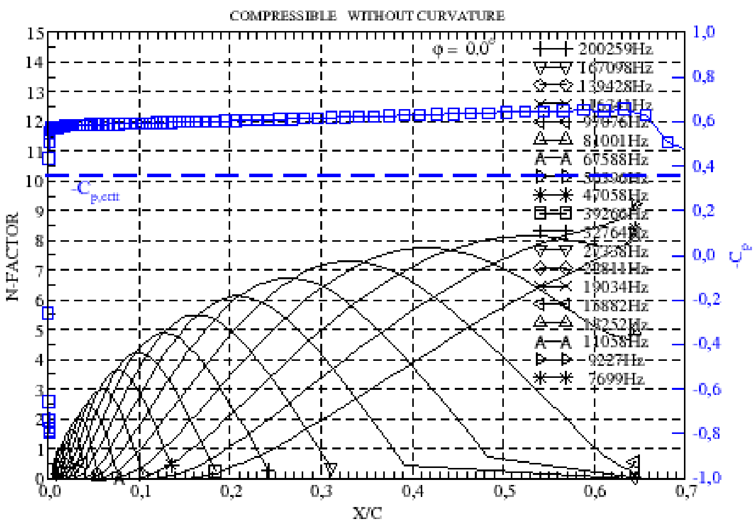
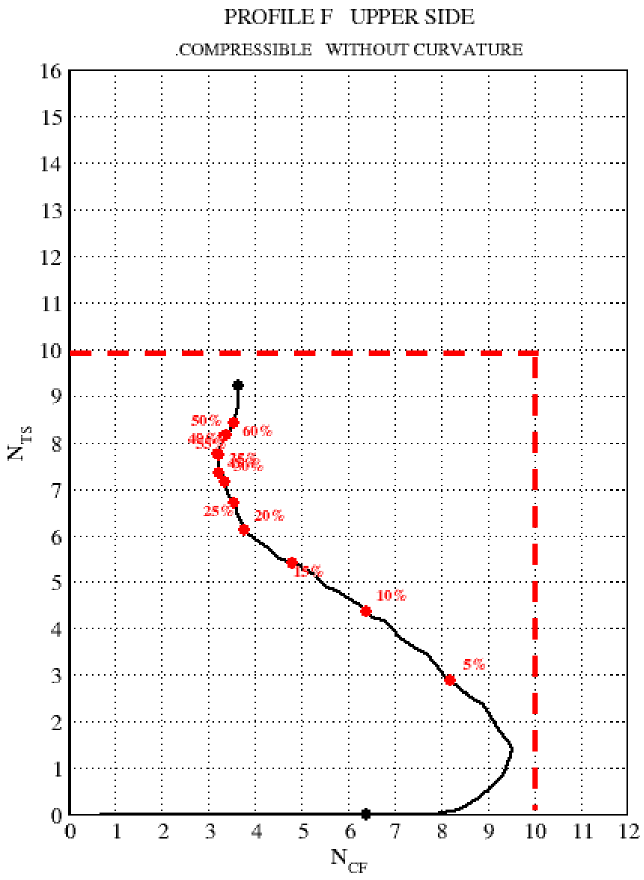
Local Mach **1.13**



Airfoil F: compressible stability theory, TS 0º degree

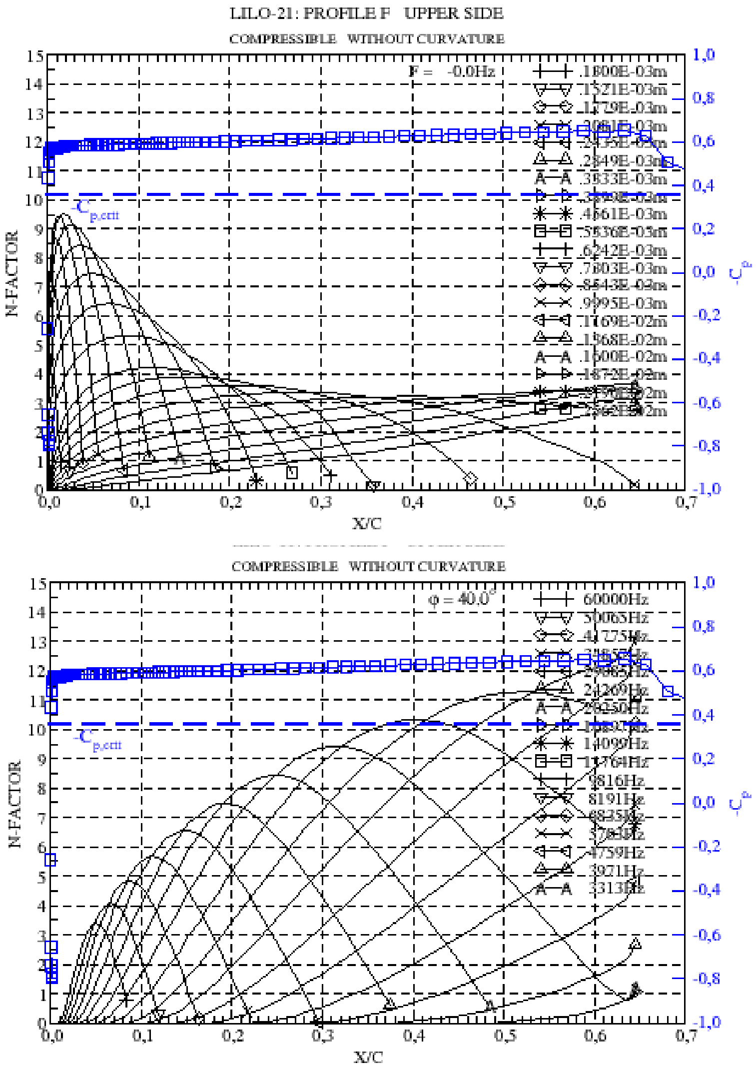


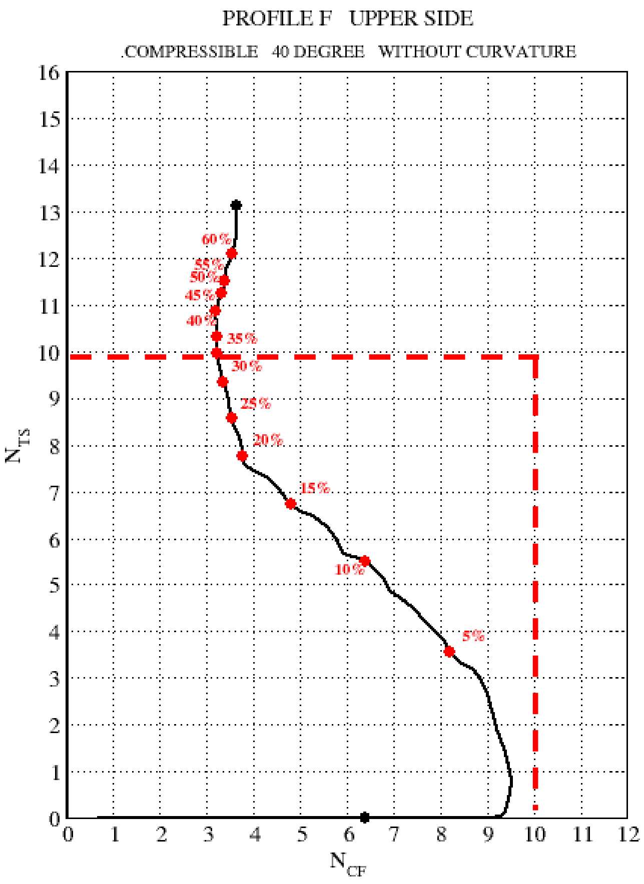




TS-transition at shock near 65%

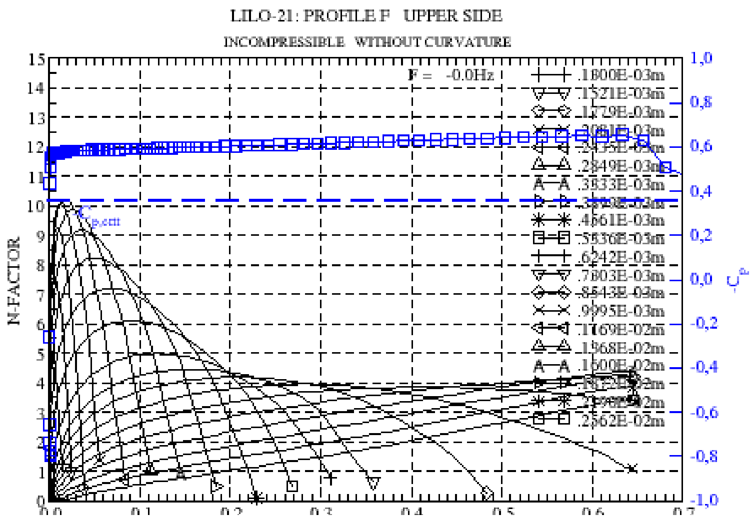
Airfoil F: compressible stability theory, TS 40º degree

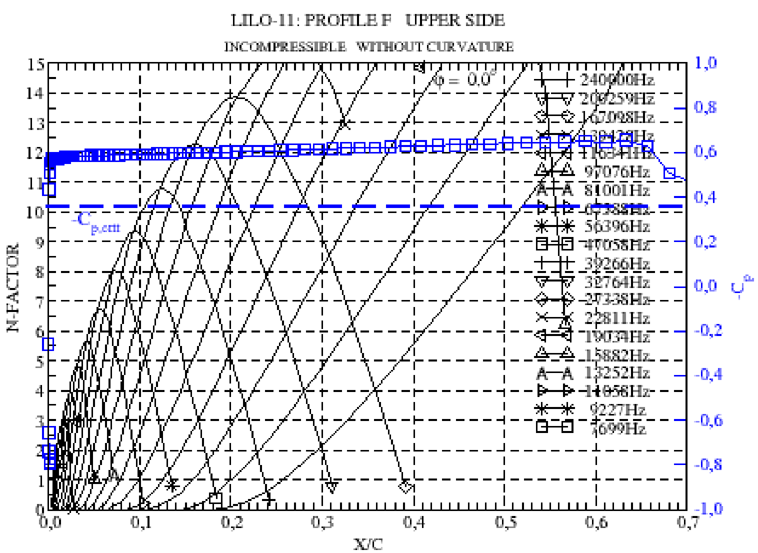


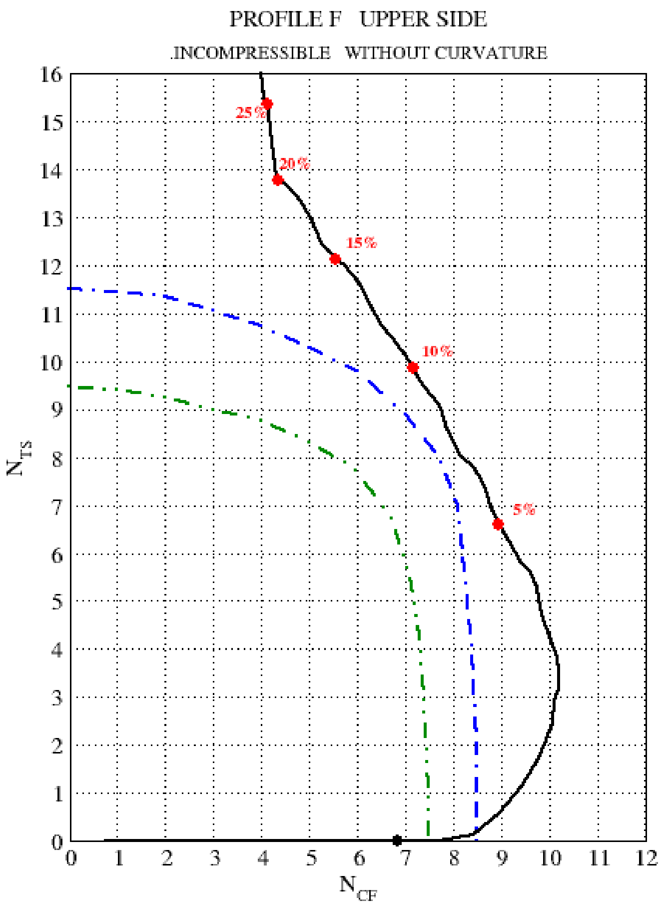


TS-transition at 33%

Airfoil F: incompressible stability theory







Early CF-transition

Airfoil F: incompressible vs. compressible LST

Compressible stability theory:

Cross-flow: 0 Hz
Tollmien-Schlichting: 0^o direction

Transition at shock. i.e. for X/C > 0.6

Compressible stability theory: most amplified Tollmien-Schlichting mode in 40^o-direction

Cross-flow: 0 Hz
Tollmien-Schlichting: 40^o direction

Tollmien-Schlichting transition at X/C = 0.33

Incompressible stability theory

Cross-flow: 0 Hz
Tollmien-Schlichting: 0^o direction

Early cross-flow transition

The stability calculations were performed with the same input boundary layer profiles, the same stability code, the same number of grid points, however, with different N-factor integration strategies (cf. chart 5) and different correlations.

Thank you for your attention

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